Free Convection in a Square Cavity with a Partially Heated Wall and a Cooled Top

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Free-convective flow in a rectangular cavity which has one "side" wall partially heated to a uniform temperature and the "top" wall cooled over its entire surface to a uniform lower temperature has been considered. All remaining wall surfaces are adiabatic. The cavity is, in general, inclined at an angle to the vertical. The flow has been assumed to be laminar and two dimensional. Fluid properties have been assumed constant except for the density change with temperature that gives rise to the buoyancy forces, this being treated by means of the Boussinesq approximation. The governing equations, expressed in terms of stream function and vorticity, have been written in dimensionless form. The resultant equations, subject to the assumed boundary conditions, have been solved using the finite-element method. Because of the possible applications that motivated the study, results have only been obtained for a Prandtl number of 0.7. Attention has been restricted to a cavity with an aspect ratio of 1, i.e., a square cavity. Results have then been obtained for Rayleigh numbers between 3,000 and 100,000 for a variety of heated wall-portion sizes and positions for angles of inclination between 0 deg and 180 deg. These results have been used to study the effects of the governing parameters on the mean heat transfer rate and these effects have been related to changes in the induced flow pattern within the cavity.

Nomenclature

A = aspect ratio, h'/w'
c = c'/w'
c' = position of center-line of heated wall section
h' = height of "vertical" walls of cavity
k = thermal conductivity
Nu = local Nusselt number based on w' for hot wall
$\overline{Nu}$ = mean Nusselt number based on w'
$\overline{Nu_L}$ = mean Nusselt number based on s' for hot wall
Pr = Prandtl number
q = local dimensionless heat transfer rate
$\bar{q}$ = mean heat transfer rate
Ra = Rayleigh number based on w'
s = s'/w'
s' = size of heated wall section
T = dimensionless temperature
T' = temperature
T'_C = temperature of cold wall
T'_H = temperature of hot wall
u' = velocity component in x' direction
v' = velocity component in y' direction
w' = width of cavity
x = dimensionless x' coordinate
x' = horizontal coordinate position
y = dimensionless y' coordinate
y' = vertical coordinate position
ϕ = angle of inclination of cavity
ψ = dimensionless stream function
ψ_{max} = maximum value of ψ in cavity
ψ_{min} = minimum value of ψ in cavity
ψ' = stream function
ω = dimensionless vorticity
ω' = vorticity

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Introduction

FREE-CONVECTIVE flow in a rectangular cavity that has one "vertical" wall partially heated to a uniform temperature, T'_H, and the "top" wall cooled over its entire surface to a uniform lower temperature, T'_C, has been considered. All remaining wall surfaces are adiabatic. The cavity is, in general, inclined at an angle to the vertical. The flow situation considered is, therefore, as shown in Fig. 1. Interest in this situation arises because it is a highly idealized model of some situations that arise in the electrical industry.

Two-dimensional free-convective flow in a rectangular cavity with one vertical wall heated to a uniform temperature and the other vertical wall cooled to a uniform temperature and with the horizontal walls adiabatic has been the subject of many numerical and some experimental studies, e.g., see Catton,[1] Ostrach,[2] Ostrach,[3] Wong and Raithby,[4] and de Vahl Davis.[5] The interest in this type of flow arises because it is widely accepted as a flow that can be used for testing and evaluating numerical solution procedures for fluid flow, e.g., see de Vahl Davis, and Jones[6] because it is an approximate model of a number of practically important flow situations. There appear to be no previous studies of the actual problem being considered here, although there have been several studies concerned with the situation in which one wall of a cavity is partially heated to a uniform temperature and the opposite parallel wall is cooled to a uniform lower temperature e.g., see Chu et al.,[7] Cesini et al.,[8] Oosthuizen and Paul,[9] Kaviany,[13] Kuhn and Oosthuizen,[14] and Shakerin et al.[15] An experimental study of heat transfer in a cavity with a vertical wall containing five heater sections and with a cooled upper surface is described by Carmona and Keyhani.[16] Because aspect ratios significantly greater than one were considered in this study and because there were multiple heated sections, the results obtained by these authors are not directly comparable to those obtained in the present study.

Governing Equations and Solution Procedure

The flow has been assumed to be steady, laminar, and two dimensional. The solution is based, of course, on the use of the steady, two-dimensional Navier-Stokes, continuity and energy equations. Fluid properties have been assumed to be constant except for the density change with temperature that gives rise to the buoyancy force, this being treated by means

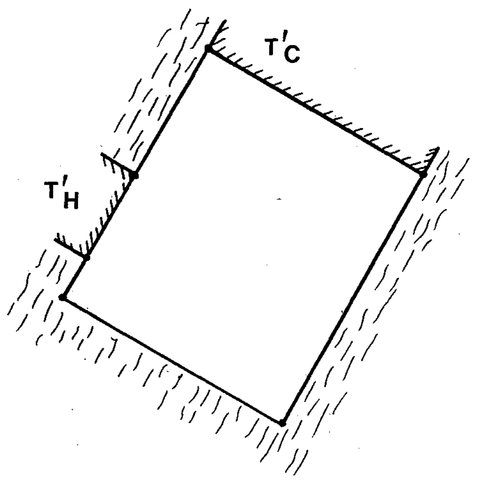

Fig. 1 Form of cavity considered.

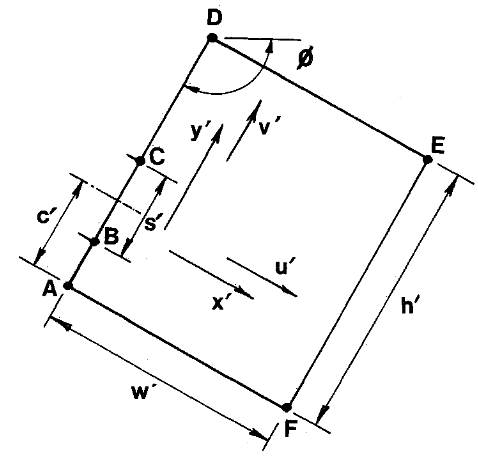

Fig. 2 Coordinate system and wall segments used in defining boundary conditions.

of the Boussinesq approximation. The governing equations are then

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$u' \frac{\partial v'}{\partial x} + v' \frac{\partial v'}{\partial y} = \beta g (T' - T'_C) \sin\phi - \frac{1}{\rho}\frac{\partial p'}{\partial y'} + \nu\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2}\right) \tag{2}$$

$$u' \frac{\partial u'}{\partial x} + v' \frac{\partial u'}{\partial y} = \beta g (T' - T'_C) \cos\phi - \frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2}\right) \tag{3}$$

$$u' \frac{\partial T'}{\partial x'} + v' \frac{\partial T'}{\partial y'} = \left(\frac{k}{\rho c}\right)\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) \tag{4}$$

The prime (′) denotes a dimensional quantity and the coordinates are as defined in Fig. 2.

The solution has been obtained in terms of the stream function and vorticity defined, as usual, by

$$u' = \frac{\partial \psi'}{\partial y'}, \qquad v' = -\frac{\partial \psi'}{\partial x'}, \qquad \omega' = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'} \tag{5}$$

The following dimensionless variables have been introduced

$$x = x'/w', \qquad y = y'/w', \qquad \psi = \psi'\mathrm{Pr}/\nu$$

$$\omega = \omega' w'^2 Pr/\nu, \qquad T = (T' - T'_C)/(T'_H - T'_C) \tag{6}$$

the cavity width, w', thus being used as the characteristic length scale.

In terms of these variables, the governing equations become

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

$$\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} = \frac{1}{Pr}\left(\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y}\right) \tag{7}$$

$$-Ra\left(\frac{\partial T}{\partial x}\sin\phi + \frac{\partial T}{\partial y}\cos\phi\right) \tag{8}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} \tag{9}$$

where Ra is the Rayleigh number based on the cavity width, w', i.e.

$$Ra = \beta g (T'_H - T'_C) w'^3/\nu\alpha \tag{10}$$

The boundary conditions on the solution are as follows: the lettered wall segments being as defined in Fig. 2,
on all walls

$$\psi = 0$$

on BC

$$T = 1 \tag{11}$$

on DE

$$T = 0$$

on all remaining surfaces

$$\partial T/\partial n = 0$$

where n is the coordinate measured normal to the surface.

The above dimensionless governing equations, subject to these boundary conditions, have been solved using the finite element method. The procedure adopted has been successfully used before in a number of studies of natural convective cavity flows, e.g., see Oosthuizen and Paul.[10-12]

The solution directly gives the local dimensionless heat transfer rate distributions on the walls of the cavity, i.e., the local Nusselt number distributions on the hot and cold walls. This Nusselt number is given by

$$Nu = \frac{qw'}{k(T'_H - T'_C)} \tag{12}$$

These distributions can then be integrated to give the mean dimensionless heat transfer rates on the hot and the cold walls. This mean heat transfer rate has been expressed in the form of a mean Nusselt number based, for the heated wall section, on the size of the heated section s' and based, for the cold wall, on the cavity width w', i.e., for the heated section in terms of

$$\overline{Nu} = \frac{\bar{q}s'}{k(T'_H - T'_C)} \tag{13}$$

and for the cold wall in terms of

$$\overline{Nu} = \frac{\bar{q}w'}{k(T'_H - T'_C)} \tag{14}$$

In these equations, $\bar{q}$ is the mean heat transfer rate for the surface being considered and the mean Nusselt numbers for the two surfaces as so defined will be equal.

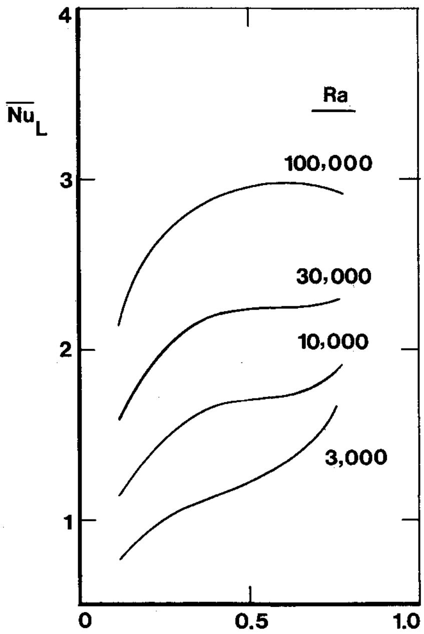

Fig. 3 Variation of mean Nusselt number based on heated element length with c for various values of Rayleigh number for s = 0.25 for an angle of inclination of 90 deg.

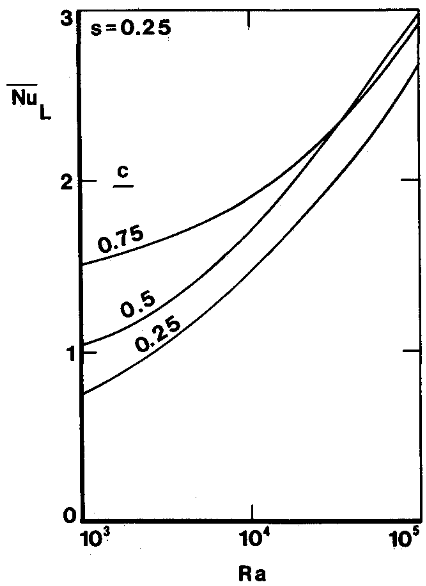

Fig. 4 Variation of mean Nusselt number based on heated element length with Rayleigh number for various values of c for s = 0.25 for an angle of inclination of 90 deg.

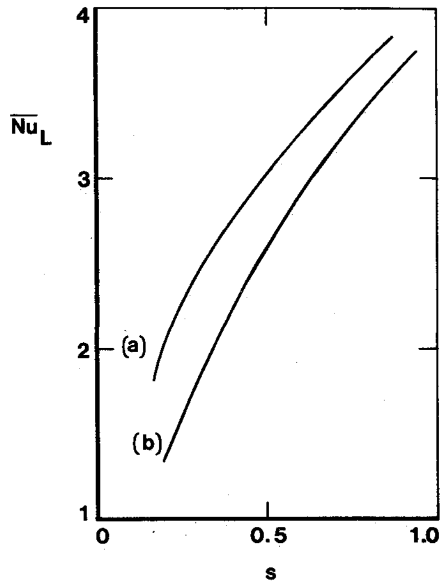

Fig. 5 Variation of mean Nusselt number, based on heated element length, at an angle of inclination of 90 deg with s for a Rayleigh number of 30,000 for a) the case of c = 0.5, i.e., the heated wall section is centered on the wall, and b) the case where the lower edge of the heated wall section is touching the bottom of the cavity.

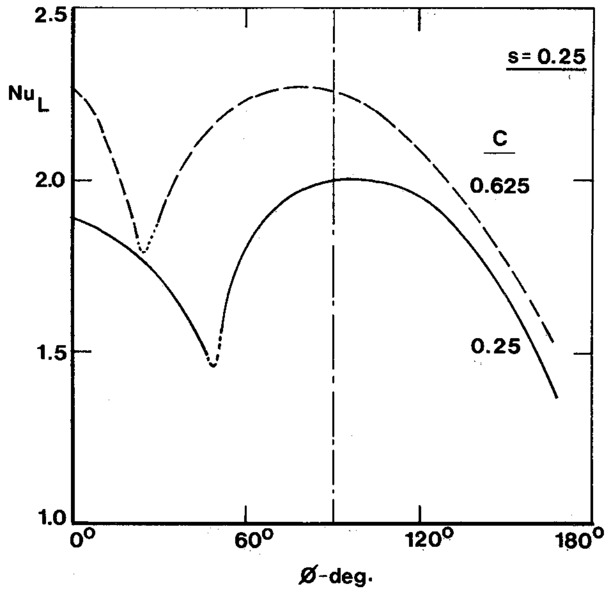

Fig. 6 Variations of mean Nusselt number, based on heated element length, with angle of inclination for c = 0.25 and a Rayleigh number of 30,000 for s values of 0.25 and 0.625.

Results

The solution to the equations given in the previous section has, in general, the following parameters: 1) the Rayleigh number, Ra; 2) the Prandtl number, Pr; 3) the size of the heated portion of the wall relative to the width of the cavity $s = s'/w'$; 4) the distance of the centerline of the heated portion of the wall along the wall relative to the width of the cavity $c = c'/w'$; and 5) the angle of inclination of the cavity, ϕ.

Because of the possible applications that motivated the present study, results have only been obtained for $Pr = 0.7$, which leaves as parameters Ra, s, c and ϕ. In general, the aspect ratio of the cavity, $A = h'/w'$, would also be a parameter but results have been obtained here only for an aspect ratio of 1.0, i.e., for a square cavity. Results have been obtained for Rayleigh numbers, Ra, of between 3000 and 100,000 for angles of inclination, ϕ, between 0 deg and 180 deg. The case

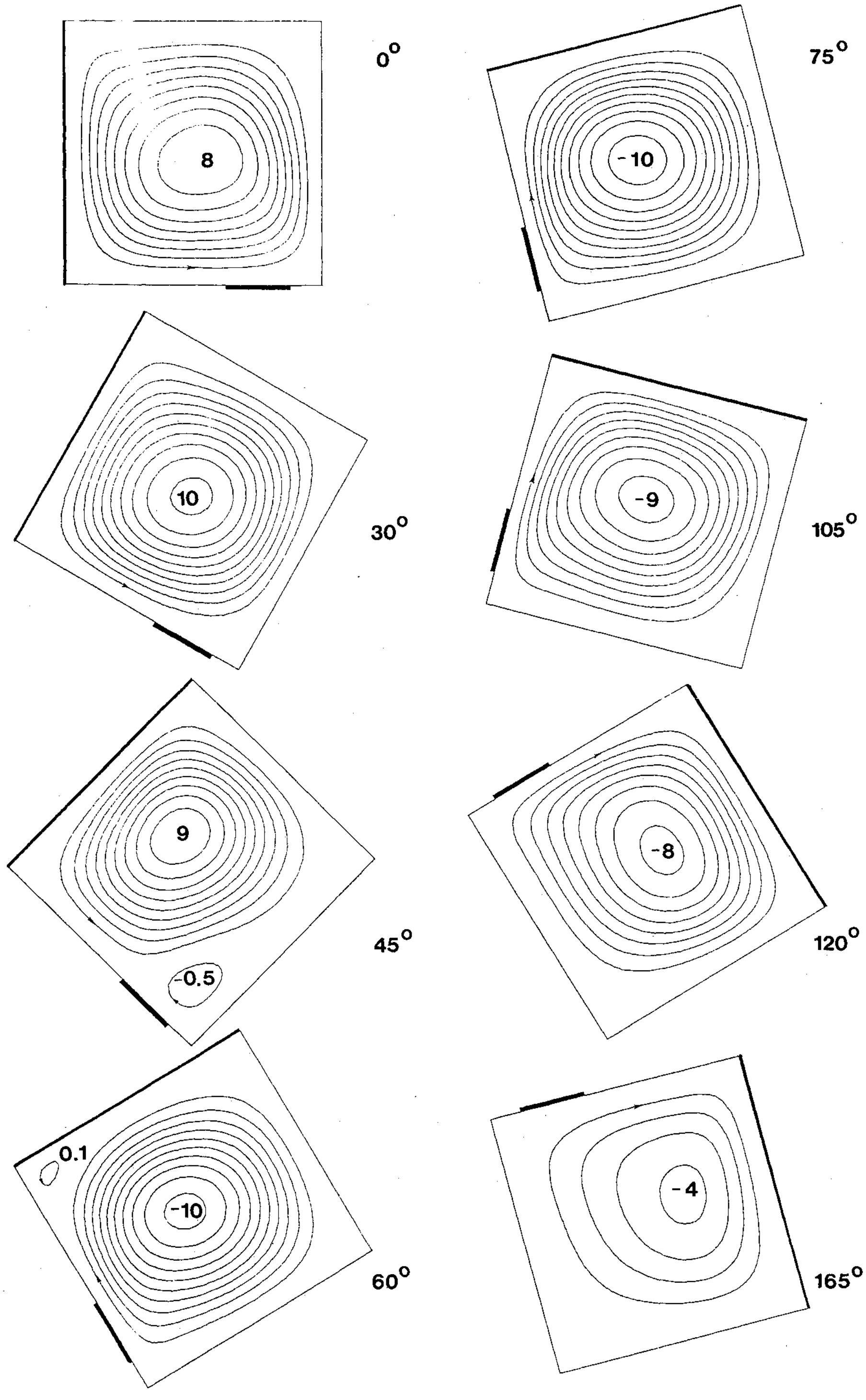

Fig. 7 Typical streamline patterns in cavity at various angles of inclination for c = 0.25 and s = 0.25 for a Rayleigh number of 30,000. Unless otherwise indicated, the stream function change between streamlines is 1.0.

of ϕ = 90 deg corresponds to the case of a vertical cavity, i.e., to the case where the wall containing the heated wall section is vertical. Because of the way in which ϕ is defined, angles between 0 deg and 90 deg correspond to the case where the wall containing the heated section is facing upwards while angles between 90 deg and 180 deg correspond to the case where this wall is pointing downwards.

Results for the vertical cavity case, i.e., for ϕ = 90 deg, will be considered first. Some typical variations of mean heat transfer for this case are shown in Figs. 3 to 6. Figure 3 shows a typical variation of mean Nusselt number for a fixed heated section size with position of heated section for various Rayleigh numbers. At low Rayleigh numbers when the heat transfer is essentially by conduction the mean Nusselt number tends to increase monotonically with increasing c, i.e., it increases continuously as the heated section approaches the cold upper surface. At high Rayleigh numbers, on the other hand, the mean heat transfer rate tends to reach a maximum when the heated element is near the center of the wall and to fall off as the element approaches the upper and lower walls. This is because at the higher Rayleigh numbers, the convective motion is dominant in determining the heat transfer rate. When the heated section is near the top or bottom walls, the convective motion tends to be roughly normal to the heated section whereas when the heated section is in the middle of the wall, the convective motion tends to be parallel to the heated surface. Similar results are shown in Fig. 4 which shows the variation of mean Nusselt number with Rayleigh number for fixed element size for various dimensionless heated section positions. It can be seen from this figure that at all positions the heat transfer rate increases with Rayleigh number but that the position for maximum heat transfer changes from near the top of the wall at low Rayleigh numbers to near the center of the wall at high Rayleigh numbers. Figure 5, curve a, shows the effect of heated element size on the mean heat transfer rate for a heated element at the center of the wall and for a fixed Rayleigh number. It will be seen that the mean Nusselt number based on the element length increases with increasing element size. Figure 5, curve b, shows similar results for a heated element that always has its lower edge at the bottom of the wall.

The effect of angle of inclination will next be considered. Figure 6 shows the variations of mean Nusselt number with angle of inclination for two heated section positions for a fixed heated section size and a fixed Rayleigh number. It can be seen that the heat transfer rate is at maximum for angles of inclination near 90 deg and decreases with increasing angle for angles greater than 90 deg. For angles less than 90 deg, the heat transfer rate first decreases with decreasing angle but then passes through a sharp minimum before rising again as an angle of 0 deg is approached. These changes in heat transfer rate are associated with changes in the flow pattern within the cavity. When the angle of inclination is near 90 deg, the flow is upward over the heated surface and then outward in the positive x-direction over the cold wall. For angles greater than 90 deg, this same flow pattern exists. As the angle is decreased below 90 deg, however, a downward flow, i.e., in the negative x-direction tends to develop over the cold wall and at small angles of inclination, this flow pattern dominates. These changes in the flow pattern are illustrated by the results given in Figs. 7 and 8. Figure 7 shows typical streamline patterns within the cavity at various angles of inclination for a fixed heated section position and size and a fixed Rayleigh number. In interpreting the results given in this figure, it should be realized that negative values of the stream function are associated with an "upward" flow over the heated section while positive values of the stream function are associated with a "downward" flow, i.e., a flow in the negative y-direction, over this heated section. It can be seen from Fig. 7, therefore, that at small angles of inclination, the flow consists of a single vortex with downward flow over the heated element. At an angle of about 45 deg, a second vortex associated with "upward" flow along the heated element is formed. With

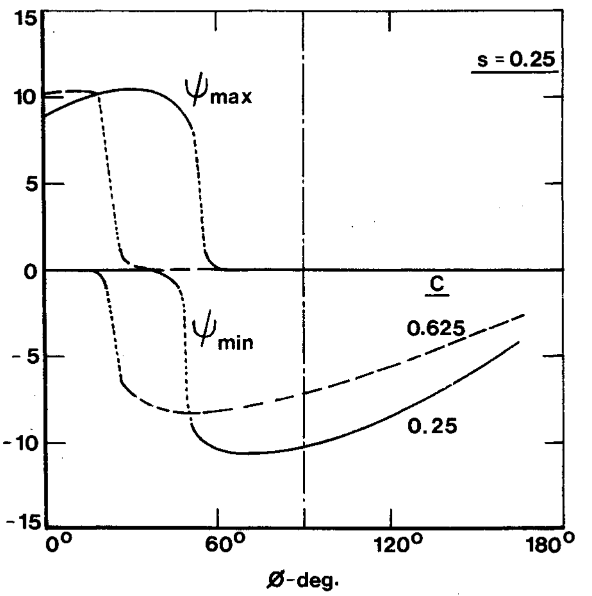

Fig. 8 Variations of maximum and minimum stream function values in cavity for c = 0.25 and a Rayleigh number of 30,000 for s values of 0.25 and 0.625.

further increase in angle of inclination, this second vortex grows rapidly in size and at angles of inclination greater than about 60 deg, the flow again consists of a single vortex. This is now being associated with "upward" flow along the heated element. These changes are further illustrated by the results given in Fig. 8. This shows the variations of the maximum and minimum values of the stream function in the cavity with angle of inclination for a fixed element size and fixed Rayleigh number for two element positions. The very sharp changes from a flow associated with a counter clockwise vortex to one associated with a clockwise vortex are illustrated by these results. It also can be seen from these results and from those presented in Fig. 6 that the angle at which the change from a counter clockwise vortex to a clockwise vortex occurs tends to decrease as the position of the heated section along the wall increases.

Conclusions

The results here obtained indicate that:

1) For a vertical cavity the mean heat transfer rate at low Rayleigh numbers increases as the position of the heated element up the wall increases. At high Rayleigh numbers, however, the mean heat transfer rate tends to be greatest when the heated section is near the center of the wall.

2) The highest mean heat transfer rate tends to occur when the angle of inclination is either near 90 deg or near 0 deg.

3) At small angles of inclination the flow in the cavity is dominated by a single vortex associated with "downward" flow over the heated section while at larger angles of inclination the flow in the cavity is dominated by a single vortex associated with "upward" flow over the heated section. The change from one type of flow to the other occurs over a narrow range of angle of inclination.

Acknowledgments

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